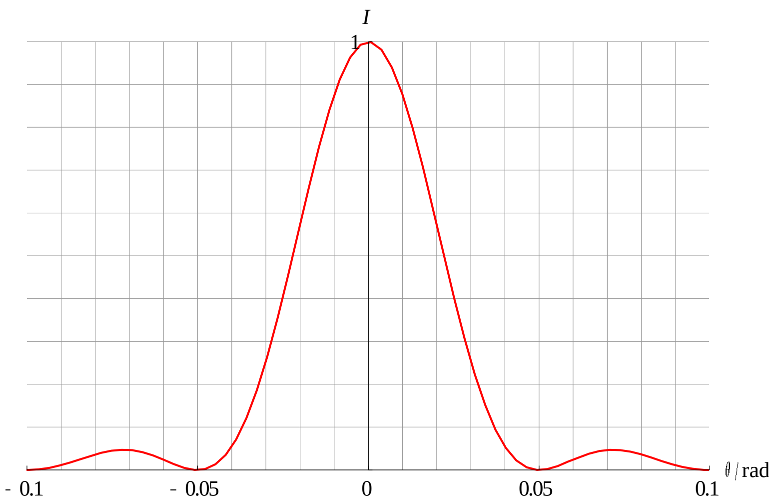
Problem of the week

Wave phenomena (HL only)

(a) The graph shows the variation of intensity with diffraction angle for single slit diffraction. The units on the intensity axis are arbitrary.



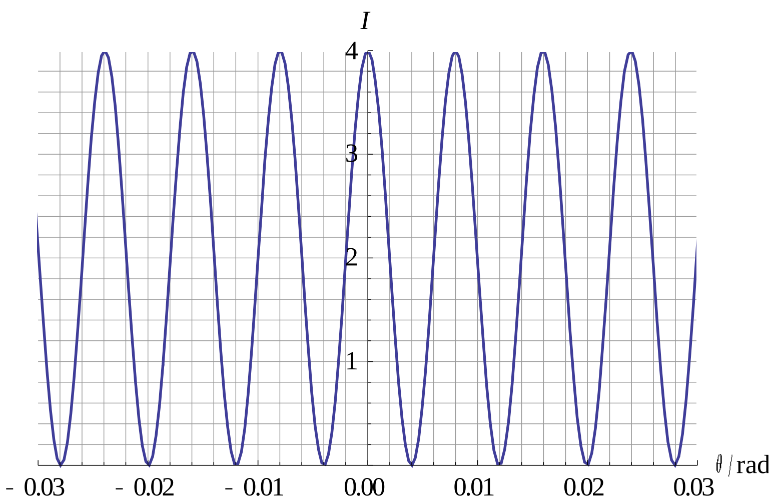
The wavelength of light is 522 nm.

(i) Describe how the central maximum at $\theta = 0$ is formed.

(ii) Determine the width of the slit.

(iii) The slit width is halved. State and explain two changes that will come about in the graph above.

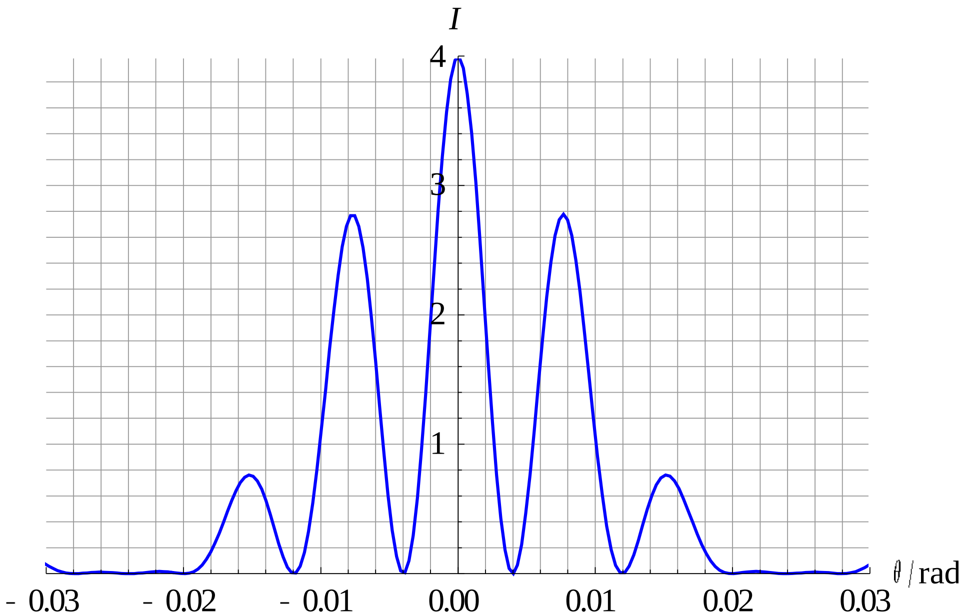
(b) The graph shows the intensity of light on a screen 0.600 m from two **very narrow** slits.



The wavelength of light is 720 nm.

Show that the separation of the slits is 9.0×10^{-5} m.

(c) The two slits in (b) are replaced by two slits of non-negligible width. Their separation stays the same. The graph shows the new variation of intensity with diffraction angle.



(i) Explain why the maxima are not of equal intensity.

(ii) Suggest why the third order maximum of the original pattern is missing.

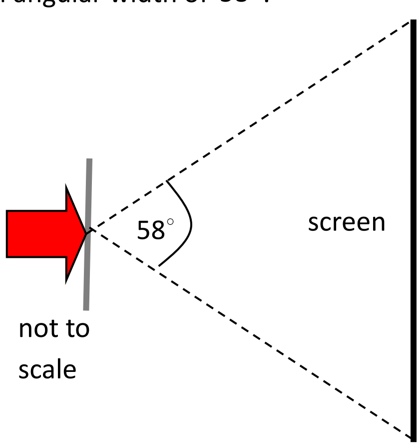
(iii) Determine the slit width.

(d) The number of slits is increased to 6 without changing the slit width and slit separation. List three changes to the pattern in (c) and state a feature that will remain the same.

(e)

(i) Monochromatic light is incident normally on a diffraction grating with 300 lines per mm. The angle separating the first and second order maxima is 14°. Determine the wavelength of light.

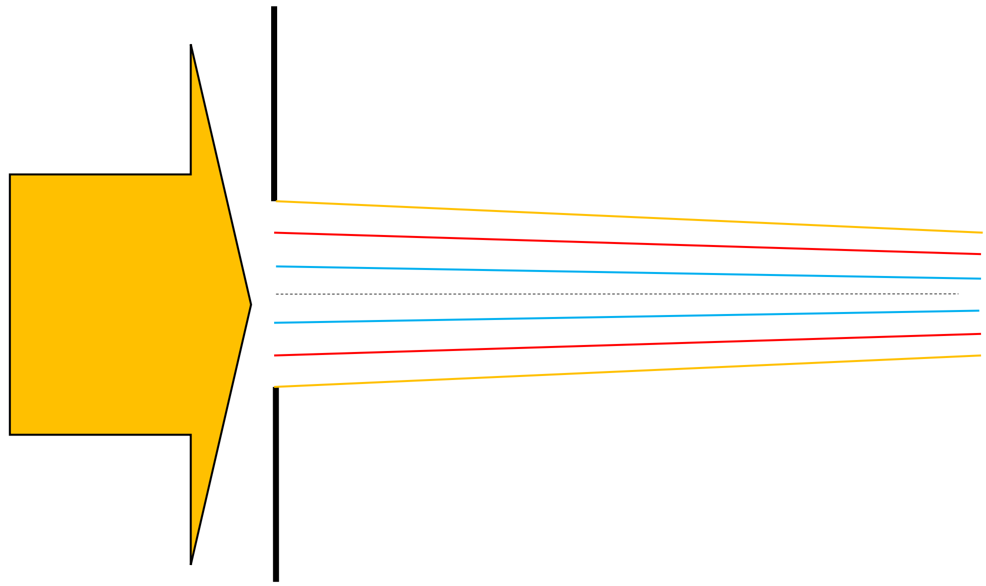
(ii) Predict the number of orders that are visible on a screen behind the grating. The screen has an angular width of 58°.



Answers

(a)

(i) The rays arriving at $\theta = 0$ may be grouped in pairs as shown in the diagram:



Pairs of rays in the same color have zero path/phase difference and so interfere constructively.

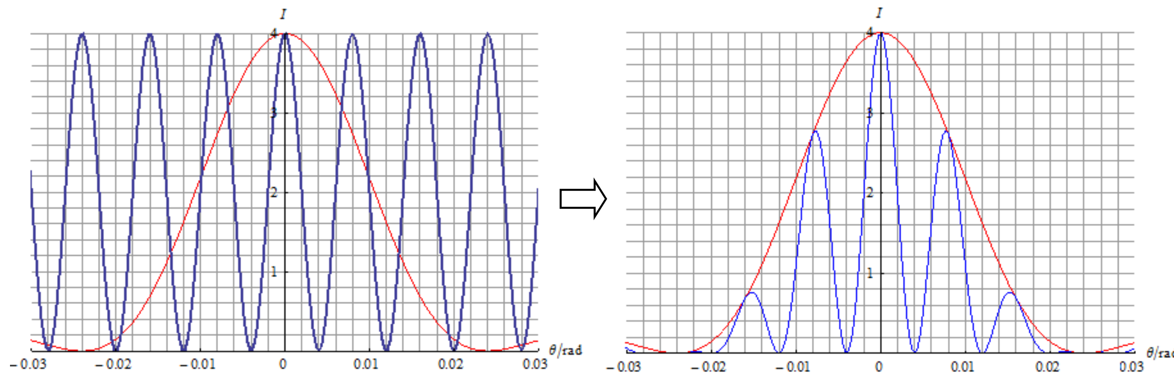
(ii) $\theta = \dfrac{\lambda}{b} \Rightarrow b = \dfrac{\lambda}{\theta} = \dfrac{522 \times 10^{-9}}{0.05} = 1.04 \times 10^{-5}$ m.

(iii) From $\theta = \dfrac{\lambda}{b}$ the angle at which the first diffraction minimum occurs will move to ± 0.1 rad. The peak intensity at $\theta = 0$ will become 4 times less. This is because half the rays go through the slit and so the amplitude at $\theta = 0$ is half that of the original case. Intensity is proportional to the square of the amplitude and so the intensity is 4 times less.

(b) $s = D\theta = 0.6 \times 0.008 = 4.8 \times 10^{-3}$ m. $s = \dfrac{\lambda D}{d} \Rightarrow d = \dfrac{\lambda D}{s} = \dfrac{720 \times 10^{-9} \times 0.600}{4.8 \times 10^{-3}} = 9.0 \times 10^{-5}$ m.

(c)

(i) The single slit pattern modulates the pattern in (b) and so the intensity of the maxima other than the central one is reduced according to the single slit pattern.



(ii) The third order maximum of the 2-slit pattern coincides with the first minimum of the single slit pattern.

(iii) The first minimum of the single slit pattern is at $\theta = 0.024$ rad and so

$$b = \frac{\lambda}{\theta} = \frac{720\times 10^{-9}}{0.024} = 3.0\times 10^{-5}\text{ m}.$$

OR

The third order maximum is observed at an angle given by $d\sin\theta = 3\lambda$ or $d\theta \approx 3\lambda$ since the angle is very small. Hence $\theta \approx \frac{3\lambda}{d} = \frac{\lambda}{b}$ and so $b = \frac{d}{3} = 3.0\times 10^{-5}$ m.

(d)

- The intensity of all maxima will increase.
- The width of each maximum will decrease.
- There will be secondary maxima of small intensity in between the primary maxima.
- The position of the primary maxima will stay the same.

(e)

(i) The second order maximum is observed at $d\sin\theta_2 = 2\lambda \Rightarrow \theta_3 = \arcsin\frac{2\lambda}{d}$. And the first at $\theta_1 = \arcsin\frac{\lambda}{d}$. Hence $\theta_2 - \theta_1 = \arcsin\frac{2\lambda}{d} - \arcsin\frac{\lambda}{d}$. This means that $\arcsin(2\lambda\times 300\times 10^3) - \arcsin(\lambda\times 300\times 10^3) = 14^\circ\times\frac{\pi}{180^\circ}$. The GDC gives the solution as $\lambda = 7.62385\times 10^{-7} \approx 7.6\times 10^{-7}$ m.

(You have spent a fortune on your GDC, why only use it in math classes?)

(ii) $d\sin 29^\circ = n\lambda \Rightarrow n = \frac{d\sin 29^\circ}{\lambda} = \frac{10^{-3}\times\sin 29^\circ}{300\times 7.62385\times 10^{-7}} = 2.12$. Hence 2+2+1 = 5 orders are visible on the screen.